B.Sc. Part I (Hons.) 1st Paper

TRIGONOMETRY (HYPERBOLIC FUNCTIONS)

Q: Separate $\sinh(\alpha+i\beta)$ and $\operatorname{sech}(\alpha+i\beta)$ into real and imaginary parts.

Soln.

$\because \sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y$.

$$\Rightarrow \sinh(\alpha+i\beta) = \sinh\alpha \cosh(i\beta) + \cosh\alpha \sinh(i\beta)$$

$$= \sinh\alpha . \cos\beta + \cosh\alpha . (i\sin\beta)$$

$$= \sinh\alpha \cos\beta + i\cosh\alpha \sin\beta.$$

Now, $$\operatorname{sech}(\alpha+i\beta) = \frac{1}{\cosh(\alpha+i\beta)}$$

$$= \frac{1}{\cos i(\alpha+i\beta)} = \frac{1}{\cos(i\alpha-\beta)}$$

$$= \frac{\cos(i\alpha+\beta)}{\cos(i\alpha-\beta)\cos(i\alpha+\beta)}$$

$$\Rightarrow \operatorname{sech}(\alpha+i\beta) = \frac{2\cos(i\alpha+\beta)}{2\cos(i\alpha-\beta)\cos(i\alpha+\beta)} = \frac{2\cos(i\alpha+\beta)}{\cos\{i\alpha-\cancel{\beta}+i\alpha+\cancel{\beta}\} + \cos\{\cancel{i\alpha}+\beta-\cancel{i\alpha}+\beta\}}$$

$$\Rightarrow \operatorname{sech}(\alpha+i\beta) = \frac{2\cos(i\alpha+\beta)}{\cos 2i\alpha + \cos 2\beta} = \frac{2(\cos i\alpha . \cos\beta - \sin i\alpha \sin\beta)}{\cosh 2\alpha + \cos 2\beta}$$

$$= \frac{2\cos i\alpha \cos\beta - 2\sin i\alpha \sin\beta}{\cosh 2\alpha + \cos 2\beta}$$

$$\Rightarrow \operatorname{sech}(\alpha+i\beta) = \frac{2\cosh\alpha\cos\beta - 2i\sinh\alpha\sin\beta}{\cosh 2\alpha + \cos 2\beta}$$

$$\Rightarrow \operatorname{sech}(\alpha+i\beta) = \frac{2\cosh\alpha\cos\beta}{\cosh 2\alpha+\cos 2\beta} - i\,\frac{2\sinh\alpha\sin\beta}{\cosh 2\alpha+\cos 2\beta}$$

Thus, $\operatorname{sech}(\alpha+i\beta)$ is separated into real and imaginary parts.

Q If $\tan(\theta+i\phi) = \cos\alpha + i\sin\alpha$, prove that $\theta = \frac{n\pi}{2} + \frac{\pi}{4}$.

Soln.

$$\tan(\theta+i\phi) = \cos\alpha + i\sin\alpha$$

$$\Rightarrow \tan(\theta-i\phi) = \cos\alpha - i\sin\alpha.$$

Now, $\tan 2\theta = \tan\{(\theta+i\phi)+(\theta-i\phi)\}$

$$= \frac{\tan(\theta+i\phi)+\tan(\theta-i\phi)}{1-\tan(\theta+i\phi)\tan(\theta-i\phi)}$$

$$= \frac{\cos\alpha+i\sin\alpha+\cos\alpha-i\sin\alpha}{1-(\cos^2\alpha-i^2\sin^2\alpha)} = \frac{2\cos\alpha}{1-1}$$

$$\Rightarrow \tan 2\theta = \infty = \tan\frac{\pi}{2}$$

$$\Rightarrow 2\theta = n\pi + \frac{\pi}{2}$$

$$\Rightarrow \theta = n\pi + \frac{\pi}{4}$$